Leibniz's Early Encounters with Descartes, Galileo, and Spinoza on Infinity.

Abstract

This paper seeks to highlight and trace some of the main threads that Leibniz used in developing his views on infinity in his early years in Paris (1672-76). In particular, I will be focusing on Leibniz's encounters with, Descartes, Galileo, and Spinoza. Through these encounters, the most significant features of Leibniz's view of infinity will begin to emerge. Leibniz's response to Descartes reveals his positive attitude to infinity. He rejects Descartes' view that, since we are finite, we cannot comprehend the infinite and therefore should refrain from studying it. Likewise, Leibniz rejects Descartes' view that the term 'infinite' should be reserved to God alone, as well as his distinction between the infinite and the indefinite. Leibniz's encounter with Galileo brings out his rejection of infinite number in response to Galileo's paradox. This, in turn, leads him to face another formidable challenge, viz., to defend the claim that an infinite being is possible, while an infinite number is not. Leibniz's encounter with Spinoza, I suggest, highlights the way he is approaching this problem by distinguishing between different senses and different degrees of infinity. The strategy of employing different senses of infinity in different contexts of his philosophy, remains central to Leibniz's approach to infinity in the rest of his career.

Introduction: the Significance of Encounters for Leibniz[1]

Since his early youth to the very end of his life, Leibniz never stopped absorbing and assimilating new information that he would quickly use in his writing. In some of his autobiographical remarks Leibniz describes how, at a very tender age, he taught himself to

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[1] Abbreviations: AG - Leibniz, G. W., *Philosophical Essays*, eds. and trans. Garber, D., and Ariew, R. (Indianapolis: Hackett, 1989); AT- Descartes, R., *Œuvres de Descartes*, 11 vols., eds. Adam, C. and Tannery, P. (Paris: J. Vrin, 1996); *Confessio* - Leibniz, G. W., *Confessio philosophi*, *Papers Concerning the Problem of Evil, 1671–1678*, ed. and trans. Sleigh, R. C., JR. (New Haven: Yale University Press, 2005). CSM - Descartes, R., *The Philosophical Writings of Descartes*, vols. 1,2, eds. and trans. Cottingham, J., Stoothoff, R. and Murdoch., D. (Cambridge: Cambridge University Press, 1985); CSMK - Descartes, René, *The Philosophical Writings of Descartes*, vol. 3, eds. and trans. Cottingham, J., Stoothoff, R., Murdoch., D. and Kenny, A. (Cambridge: Cambridge University Press, 1991); Curley - Spinoza, B. de: *Collected Works,* vol. 1, ed. and trans. Curley, E. (Princeton: Princeton University Press, 1988); DSR - Leibniz, G. W., *De Summa Rerum: Metaphysical Papers 1675-1676* trans. and ed. Parkinson G. H. R. (New Haven and London: Yale University Press, 1992); E- Spinoza, B. de, "Ethics"; EN - Galileo, Galilei. *Opera*. Edizione Nazionale, ed. Favaro, A. (Florence, 1898); Gebhardt - Spinoza B., *Opera*. 4 vols, ed. Gebhardt, C. (Heidelberg: Carl Winter, 1925); L - Leibniz, G. W., P*hilosophical Papers and Letters,* ed. and trans Loemker, L. (Dordrecht: Reidel, 1969); LLC - Leibniz, G. W., *The Labyrinth of the Continuum. Writings on the Continuum Problem, 1672-1686*, ed. and trans. Arthur, R. (New Haven and London: Yale University Press, 2001); NE - Leibniz, G. W., *Nouveaux essais sur l'entendement humain,* trans. and eds. Remnant, P. and Bennett, J.(Cambridge: Cambridge University Press, 1981, 2d ed. 1996), cited by book, chapter and section.

read Latin through an illustrated edition of Livy's Roman history that he found in his father's library. Once allowed into the library, he went through his father's books with great curiosity and pleasure.[2]

Leibniz's vast erudition is clearly one of his distinctive traits. At the same time, he was not a typical erudite, a scholar with a gift for accumulating and memorizing what he reads. Leibniz was also gifted with a rare ability to quickly adapt and modify what he read. It seems as though what he reads immediately becomes useful resource in his productive mind, material that he used with great ease, facility, and originality. It is no wonder therefore that Leibniz has often been described as an eclectic. But Leibniz was not an eclectic who simply borrows from diverse sources. Rather, his approach to his many sources marks a definite pattern. In fact, one of Leibniz's most distinctive intellectual traits – and one of his most remarkable talents – was an ingenious ability to integrate, synthesize, and reconcile views that seem far a part, or even opposed. By reconciling views that seem wide apart, Leibniz was producing original views that bear the mark of his synthetic and creative mind.[3]

His eagerness to reconcile and integrate multiple sources was further motivated by his conviction that each view contains (at least) a part of the truth. As he tells Remond in 1714: "I have found that most of the sects are right in a good part of what they propose, but not so much in what they deny... I flatter myself to have penetrated into the harmony of these different realms and to have seen that both sides are right provided that they do not clash with each other,..." (GP III 606; L 654-5).

Thus, as Leibniz saw this, the most important task was to produce a framework that would maximize the coherence of (or, more precisely, would harmonize) apparently conflicting views. It is worth noting that, according to Leibniz, harmony itself is seen as unity in diversity. The epistemic sense of harmony – a sense of forming a unified view of various sources – would ultimately track the harmony, which, according to Leibniz, was the very principle of creating the world (and thus the harmony among created things). To produce harmony among diverse views – that is, to achieve order and unity among different views and various disciplines – was clearly one of Leibniz's central aims.

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[2] Although his autobiographical account seems rather incredible, it is probably only slightly exaggerated. For a, balanced, and well-informed account of Leibniz's intellectual formation, see Antognazza, M. R., *Leibniz: An Intellectual Biography* (Cambridge: Cambridge University Press, 2009), especially pp. 30-37. While most accounts highlight Leibniz as a miraculous autodidact, Antognazza's account balances this with his formal schooling: "At school he was drilled for hours in textbook fashion on a tiny canon of established authorities in a restrictive elementary curriculum. At home he spent entire days wandering freely in the intellectual terra incognita of this father's library..." (35).

[3] One early example of Leibniz's intellectual attitude is the construction of his early vision in the project for catholic demonstrations and his 1668 *confessio Naturae contra Atheistas*. See next section for more details and footnote 7, in particular.

As much as Leibniz was eager to read and learn, so he was eager to compose and exchange. He surely was one of the most prominent citizens of the "Republic of Letters". His lively and voluminous correspondence testifies for the dialogical character of his thinking and the way he engages with other thinkers' work.[4] Precisely because Leibniz read so much and conversed with so many, pointing to *his ultimate* sources and *the* readings that shaped his views is not only impossible but probably a misguided approach. And yet, for the very reasons noted above, especially because Leibniz is always engaging with works of others through his conciliatory approach, looking at Leibniz's intellectual encounters is immensely rewarding. In fact, in my view, there is no better way to grasp the subtlety and complexity of his views, as long as we keep in mind that we shall not exhaust the sources that figure in his work and that his approach to his sources is creative and productive: serving the formation of his own views. It is with this in mind that I propose to present his early views on infinity by exploring some of the (many) encounters that shaped his views. In what follows I propose to present Leibniz early views of infinity as stemming from his engagement with Descartes, Galileo, and Spinoza.

Encounter is a loose enough word for my purposes here. I use it to designate Leibniz's engagement with the texts and views of several major thinkers. I do not attempt here a thorough presentation of Leibniz's exchanges; rather, I seek to highlight and trace some of the main threads that Leibniz used in developing his views on infinity. In particular, in this paper, I will be focusing on Leibniz's encounters with, Descartes, Galileo, and Spinoza. Through these encounters, the most important features of Leibniz's view of infinity will begin to emerge. Leibniz's response to Descartes reveals his positive attitude to infinity. He rejects Descartes' view that, since we are finite, we cannot comprehend the infinite and therefore should refrain from studying it. Likewise, Leibniz rejects Descartes' view that the term 'infinite' should be reserved to God alone. Leibniz's encounter with Galileo brings out his response to Galileo's paradox and his rejection of infinite number — in response to Galileo's paradox. This in turn leads him to face another formidable challenge, viz., to defend the claim that an infinite being is possible, while an infinite number is impossible. Leibniz's encounter with Spinoza, I suggest, highlights his way of approaching this problem by distinguishing between different senses and different degrees of infinity. This strategy, employing different senses and different degrees of infinity in different contexts of his philosophy, remains at the heart of Leibniz's approach to infinity for the rest of his career.

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[4] One of the most telling facts illustrating Leibniz's attitude, apart from the sheer volume of his letters, is his composition of the imaginary dialogue with Locke, resulting in the "New Essays on Human Understanding," due to his frustration from Locke lack of response to his attempts to draw him into dialogue.

Leibniz Encounter with Descartes: the Infinite, the Indefinite, and the Attitude Towards Infinity

When Leibniz made his first steps into the scholarly world, Descartes' fame was already well established, and his work widely disseminated throughout Europe. It would have been surprising, therefore, if Leibniz, who was keen to read whatever he could, and especially of the new philosophers, had not been familiar with Descartes' work. However, as Maria Rosa Antognazza notes in her recent study of Leibniz's intellectual biography, "although [Leibniz] was obviously familiar with Descartes's philosophy, his knowledge of it up to [1675] had been basically second-hand. During the winter of 1675–6 and the spring of 1676 he plunged into a careful reading of Descartes' *Principia Philosophiae* (Amsterdam 1644), leaving after him a trail of notes."[5]

One thing that would not fail to draw Leibniz's attention was Descartes' distinction between the infinite and the indefinite. In Part I, articles 26–27, of his *Principles of Philosophy*, Descartes distinguishes between the infinite and the indefinite (*infini vs indéfini*), and argues that we should not seek to comprehend the infinite, but should rather consider what we find without limits to be *indefinite* (article 26, title).[6] Descartes further argues that, since we are finite beings, we should avoid discussing the infinite and thus avoid the paradoxes surrounding it.

> "this is why we should not concern ourselves to respond to those who ask if half of an infinite line is infinite, and whether an infinite number is even or odd, and other similar things, because only those who imagine that that their spirit is infinite have to examine such difficulties."[7]

Descartes further argues that we should reserve the term "infinite" for God alone, for only God's nature can be properly called infinite. All other things that we perceive to have no limits, such as the extension of the universe or the number of the stars, should be regarded as indefinitely large. Descartes further argues that their indefiniteness does not belong to their nature but rather stems from the fact that human understanding is limited and

[5] Antognazza, 2009, 167.

[6] For a translation of Leibniz's comments on this article, see LLC 25.

[7] «*c'est pourquoi nous ne nous soucierons pas de répondre à ceux qui demandent si la moitié d'une ligne infinie est infinie, et si le nombre infini est pair ou non pair, et autres choses semblables, à cause qu'il n'y a que ceux qui s'imaginent que leur esprit est infini qui semblent devoir examiner telles difficultés*» (Article 26, CSM I 201–2; AT VIIIA 14–15).

deficient, and therefore cannot perceive the infinite (article 27).[8]

Leibniz's note on Descartes' distinction (articles 26 and 27) reads as follows:

> "Instead of 'infinite', he recommends that we use the term 'indefinite', i.e. that whose limits cannot be found by us, and that the term "true infinity" should be reserved to God alone. But contrary to this, in Part 2, article 36, matter is admitted to be really divided by motion into parts that are smaller than any assignable, and therefore actually infinite." (A 6.3 214; LLC 25)

It is interesting to observe that, already in his early "Theory of Abstract Motion" of 1671, Leibniz sharply objects to Descartes' distinction between the infinite and the indefinite along a similar line of reasoning, that is, by defending the actual divisibility of the continuum. He writes,

> "There are actually parts in the continuum, contrary to what the most acute Thomas White believes, and *these are actually infinit*e, for Descartes's "indefinite" is not in the thing, but in the thinker." (Winter 1670–1671, A 6.2 264; LLC 339)

From this note we learn that, even before he had direct access to Descartes' *Principles of Philosophy*, Leibniz criticized Descartes for grounding the distinction between the infinite and the indefinite epistemologically, that is, in human limitations for comprehending infinity. Both remarks (of 1671 and of 1675) suggest that Leibniz is not wary of endorsing the actual division of matter to infinity. Moreover, his 1675 remark suggests that he thinks that Descartes, too, endorses something like the actual division of matter but that he uses the evasive terminology of indefinite division, so as not to acknowledge it. Note, too, that Leibniz already recasts Descartes' position in his own terms: rather than referring to the

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[8] « 2*6. Qu'il ne faut point tâcher de comprendre l'infini mais seulement penser que tout ce en quoi nous ne trouvons aucunes bornes est indéfini.*
Ainsi nous ne nous embarrasserons jamais dans les disputes de l'infini ; d'autant qu'il serait ridicule que nous, qui sommes finis, entreprissions d'en déterminer quelque chose, et par ce moyen le supposer ni en tâchant de le comprendre ; c'est pourquoi nous ne nous soucierons pas de répondre à ceux qui demandent si la moitié d'une ligne infinie est infinie, et si le nombre infini est pair ou non pair, et autres choses semblables, à cause qu'il n'y a que ceux qui s'imaginent que leur esprit est infini qui semblent devoir examiner telles difficultés. Et, pour nous, en voyant des choses dans lesquelles, selon certains sens, nous ne remarquons point de limites, nous n'assurerons pas pour cela qu'elle soient infinies, mais nous les estimerons seulement indéfinies. Ainsi, parce que nous ne saurions imaginer une étendue si grande que nous ne concevions en même temps qu'il y en peut avoir une plus grande, nous dirons que l'étendue des choses possibles est indéfinie ; et parce qu'on ne saurait diviser un corps en des parties si petites que chacune de ses parties ne puisse être divisée en d'autres plus petites, nous penserons que la quantité peut être divisée en des parties dont le nombre est indéfini ; et parce que nous ne saurions imaginer tant d'étoiles que Dieu n'en puisse créer davantage, nous supposerons que leur nombre est indéfini, et ainsi du reste.
27. Quelle différence il y a entre indéfini et infini.
Et nous appellerons ces choses indéfinies plutôt qu'infinies, afin de réserver à Dieu seul le nom d'infini ; tant à cause que nous ne remarquons point de bornes en ses perfections, comme aussi à cause que nous sommes très assurés qu'il n y en peut avoir. Pour ce qui est des autres choses, nous savons qu'elles ne sont pas ainsi absolument parfaites, parce qu'encore que nous y remarquions quelquefois des propriétés qui nous semblent n'avoir point de limites, nous ne laissons pas de connaître que cela procède du défaut de notre entendement, et non point de leur nature. »

"indefinitely divisible" he uses the phrase (which I suspect he adapts from Hobbes) "smaller than any assignable" — a phrase, which, for him, implies the syncategorematic sense of infinity. Against Descartes' attempt to reserve the use of infinity for God alone, Leibniz would hold that there are many things other than God that can be adequately seen as (and called) infinite.

In spite of Leibniz's critique of Descartes' distinction between the infinite and the indefinite, it is important to observe that there is one issue on which they agree: for both, the infinity of God is absolute, and, in the terms we are using here, for both, the infinity of God is not seen as a quantitative kind of infinity. In other words, for both Leibniz and Descartes, the infinity of God does not relate to greatness in magnitude. In a letter to Henry More, Descartes makes this point rather explicitly:

> "God is the only thing I positively understand to be infinite. As to other things like the extension of the world and the number of parts into which matter is divisible, I confess I do not know whether they are absolutely infinite; I merely know that I know no end to them, and so, looking at them from my own point of view, I call them indefinite."[9]

In his second letter to More, Descartes writes:

> "I say . . . that the world is indeterminate or indefinite, because I do not recognize in it any limits. But I dare not call it infinite as I perceive that God is greater than the world, not in respect to His extension, because, as I have already said, I do not acknowledge in God any proper [extension], but in respect to His perfection."[10]

The infinity of God, according to Descartes, relates primarily, and perhaps exclusively, to his perfection. With respect to the infinity of God, Leibniz's view is very similar to Descartes'. God's infinity does not pertain to extension or to any magnitude or other quantitative feature; rather, God's infinity pertains exclusively to perfection. It goes without saying that, for both Descartes and Leibniz, God is defined (in accordance with the tradition) as the most perfect being (*Ens Perfectissimum*). As Leibniz states, "The absolute is prior to the limited". "And just so the unbounded is prior to that which is has a boundary [terminus], since the boundary is something added" (A 6.3 502; A 6.3 392; GP I 224). And, as Robert Adams clarifies, "Leibniz's conception of divine perfection commits him to agree with Descartes that, in its own nature, the divine infinity or perfection is primitive — that it is unanalyzable and not a negation of the finite. For him, as for Descartes, the infinite, in

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[9] Descartes to Henry More, 5 February, 1649, CSMK 364.

[10] Descartes' second letter to Henry More, 15, May 1649, quoted by A. Koyré in his *From the Closed World to the Infinite Universe* (Baltimore: John Hopkins Press, 1957).
1957, p.122 (and not translated by CSMK). In a letter to Clerselier from 23 April 1649, Descartes explains, "[b]y 'infinite substance' I mean a substance which has actually infinite and immense, true and real perfections" (AT V 355; CSMK 377).

properties capable of infinity, is the primary case, and the finite is formed by limitation, or partial negation, of the infinite (NE 157f)" (Adams 1994, p. 116[11]).

But Leibniz's agreement with Descartes ends here. For Descartes states that there is no other thing that we should qualify as infinite – that we should refrain from ascribing infinity to things which seem unbounded to us because, in the final analysis, we cannot comprehend what infinity means. Further, Descartes argues that finite beings (such as we are) should not pretend to understand, or even try to understand, something infinite. Leibniz's attitude is almost the inverse. For Leibniz, there are many things (series, worlds, individuals) that can be (and, in fact, must be) understood as infinite. However, in describing all these things as infinite, Leibniz is working with different senses of infinity. And, as noted, he agrees with Descartes that only God is infinite in the *absolute* and non-quantitative sense.

One might argue that this disagreement between Leibniz and Descartes is merely about words, and that, in the end, there is no substantial difference in their positions. It is arguable that Leibniz's distinction between different senses of infinity comes down to something quite similar to Descartes' distinction between the infinite and the indefinite. Indeed, in his article, "Leibniz on the Indefinite as Infinite" (1998), Bassler[12] argues that this is indeed the case. According to Bassler, in Leibniz's notes from 1676, one finds a distinction that is very similar to Descartes'. Bassler observes (on p. 850) that Leibniz approves of the indefinite progression of natural numbers and rejects the notion of an infinite number of (finite) numbers. This observation has some basis in the texts, but Bassler's assimilation of Leibniz's notion of (the syncategorematic) infinite with Descartes' notion of the indefinite blurs some important differences between their views. Bassler argues that, in his later work, "Leibniz takes the indefinite as infinite" (p. 852). However, it is clear that Leibniz himself thought that his disagreement with Descartes was not merely terminological but rather substantial.

Notice first that the distinction Bassler is referring to is drawn within the realm of mathematics. Here, Leibniz uses the notion of infinite number as an illustration of something impossible, for an infinite number cannot be conceived and thus has no consistent notion. Yet, Leibniz qualifies as infinite some other things, which are not impossible. An infinite series is one obvious example. Leibniz sees an infinite series as possible because he defines a series through its generation rule (or law of the series) and not as a sum of its constituents.

Second, as already noted, Leibniz rejects Descartes' view that the distinction between infinite and indefinite is due to the incapability of our (finite) mind to understand the

[11] Adams, R.M., *Leibniz: Determinist, Theist, Idealist* (New York: Oxford University Press, 1994).

[12] Bassler, O. B., "Leibniz on the Indefinite as Infinite", *The Review of Metaphysics*, 51 (1998):849-79.

infinite. One obvious reason for this conviction is his mathematical work during his years in Paris. His work on infinite series and the calculus shows that he sees both the notion of infinitesimally small as well as that of infinitely large as mathematically manageable and indeed very useful. But it should be noted that Leibniz's response to Descartes precedes his development of the calculus. And thus it seems clear that his early commitment to investigate the infinite does not depend on his mathematical work. In addition, Leibniz holds that, although we certainly do not fully comprehend the infinite, we can nevertheless demonstrate some things about it. This point is clearly expressed in a letter to Malebranche from 1679:

> "The number of all numbers implies a contradiction, which I show thus: to any number there is a corresponding number equal to its double. Therefore the number of all numbers is not greater than the number of even numbers, i.e. the whole is not greater than its part. It is no use responding that our finite mind cannot comprehend the infinite, for we can demonstrate something about what we do not comprehend. And here we comprehend at least the impossibility, if this only means that there is a certain whole which is not greater than its part."[13]

Leibniz's conclusion from the above reasoning is that an infinite sum of parts, seen as a whole, is an impossible notion. But this negative result has some positive implications: it leads Leibniz, in contrast to Descartes, to make positive observations about the infinite. According to Leibniz, we can say that there are infinitely many things or parts of matter as long as we do not see them as a single whole or as a true unity. As early in 1672, Leibniz observed that "[t]here is no maximum in things, or what is the same, the infinite number of all unities is not one whole, but is comparable to nothing" (A 6.3 98; LLC 13)[14]. Thus, for Leibniz, it would be misguided to reduce infinity to something that we call undefined or

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[13] Leibniz to Malebranche*,* 22 June 1679; GP I 338, translation in Brown, G., "Leibniz's Mathematical Argument against a Soul of the World", *British Journal for the History of Philosophy*, vol. 13, no. 3 (2005): 449–88, on p. 479. This point comes up in other passages as well: "At last a certain new and unexpected light shined from where I least expected it, namely, from mathematical considerations on the nature of infinity. For there are two labyrinths of the human mind, one concerning the composition of the continuum, and the other concerning the nature of freedom, and they arise from the same source, infinity. That same distinguished philosopher I cited a short while ago preferred to slash through both of these knots with a sword since he either could not solve the problems, or did not want to reveal his view. For in his *Principles of Philosophy* I, art. 40- 41, he says that he can easily become entangled in enormous difficulties if we try to reconcile God's preordination with freedom of the will; but, he says, we must refrain from discussing these matters, since we cannot comprehend God's nature. And also, in *Principles of Philosophy* II, art. 35, he says that we should not doubt the infinite divisibility of matter even if we cannot grasp it. But this is not satisfactory, for it is one thing for us not to comprehend something, and quite something else for us to comprehend that it is contradictory" (1689? "On Freedom"; AG 95). See also: "having contented himself with saying that matter is actually divided into parts smaller than all those we can possible conceive, [Descartes] warns that the things he thinks he has demonstrated ought not to be denied to exist, even if our finite mind cannot grasp how they occur. But it is one thing to explain how something occurs, and another to satisfy the objection and avoid absurdity." ("Pacidius to Philalethes", 29 Oct.–10 Nov. 1676, A 6.3 554; LLC 183–185)

[14] See also NE 2.17.

undetermined because we cannot comprehend it. Indeed, for Leibniz, there is no categorematic infinity of things. But the notion of infinity is extremely useful .As he notes in his piece from 1676, "Infinite Numbers":

> "we conclude finally that there is no infinite multiplicity, from which it will follow that there is not an infinity of things either. Or it must be said that an infinity of things is not one whole, i.e. that there is no aggregate of them."[15]

Leibniz's conclusion here (at 1676) is that one can talk about infinitely many things as long as one does not regard these things as a totality or as making up a single whole (that would also admit of parts).[16] This is an important point that Leibniz firmly holds for the rest of his career. As an example, consider this passage from Leibniz's letter to Bernoulli of 1699:

> "Given infinitely many terms, it does not follow that there must be an infinitesimal term . . . . I concede the infinite multiplicity of terms, but this multiplicity does not constitute a number or a single whole. It signifies nothing but that there are more terms than can be designated by a number. Just so, there is a multiplicity or complex of numbers, but this multiplicity is not a number or a single whole."[17]

As we have already seen earlier, Leibniz denies the possibility of infinite quantities. But this, he thinks, need not prevent us from using infinity. For to refer to infinitesimals or infinite series is not to refer to true wholes or to true entities. In other words, one need not suppose the existence of an infinitely small (or large) quantity (or entity) in order to use the infinitesimal calculus (or to apply infinity more generally).

Since Leibniz's critical response to Descartes' distinction between the infinite and the indefinite appears in early in his work (1671), it seems that Leibniz held this approach even before he started his serious work in mathematics (under the guidance of Huygens in Paris). Indeed, not only did Leibniz hold the position before developing the calculus, but have been a partial cause for his *developing* the calculus. While such a claim seems to read much of the later development into Leibniz's early comment, Leibniz's early comment does point to some of the intuitions that might have led him to *develop* his calculus.[18]

Be this as it may, my main point here is that Leibniz's approach to investigating the infinite stands in stark contrast to Descartes'. Descartes recommends avoiding any

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[15] April 10, 1676, "Infinite Numbers", A 6.3 503; LLC 101.

[16] See also the "Conversation of Philarete and Ariste" in AG 267; GP VI 592.

[17] 21 Feb. 1699, Leibniz to Bernoulli; GM III 575; L 514, translation is from Levey, S., "Leibniz's Constructivism and Infinitely Folded Matter", in *New Essays on the Rationalists,* eds. Gennaro R., and Huenemann, C. (New York: Oxford University Press, 1999), pp. 134-62, on p. 139.

[18] See Arthur in LLC xxxiii.

discussion of the infinite and, especially, pretending that we can comprehend it. As Descartes writes to Mersenne,

> "I have read M. Morin's book. Its main fault is that he always discusses the infinite as if he had completely mastered it and could comprehend its properties. This is an almost universal fault which I have tried carefully to avoid." (28 Jan. 1641, Descartes To Mersenne; AT III 293; CSMK 171–172)

Descartes adds,

> "I have never written about the infinite except to submit myself to it and not to determine what it is or what it is not."[19]

Leibniz's attitude towards the question of infinity could not be more different. As we shall see, unlike Descartes, Leibniz does attempt to provide a positive account of the infinite and the productive ways in which it can be used in mathematics as well as in metaphysics. Leibniz's positive approach to infinity has to be considered in light of his response to Galileo.

Leibniz Encounter with Galileo

Galileo's Paradox and Leibniz's Response

The following note on Galileo's *Two New Sciences* encapsulates Leibniz's response to the paradoxes presented by Galileo:

> "Among numbers there are infinite roots, infinite squares, infinite cubes. Moreover, there are as many square numbers as there are numbers in the universe. Which is impossible. Hence it follows either that in the infinite the whole is not greater then the part, which is the opinion of Galileo and Gregory of St. Vincent, and which I cannot accept; or that infinity itself is nothing, i.e. that it is not one and not a whole." (Fall 1672, A 6.3 168; LLC 9)

In his *Discourses and Mathematical Demonstrations Concerning the Two New Sciences*, Galileo presents several paradoxes concerning the infinite. He opens the discussion with a geometrical example of the turning wheel (sometimes referred to as *rota Aristotelis,* LLC 432). Galileo is interested in comparing the lengths of the lines drawn by the perimeter as the wheel is turning with that drawn by the point at wheel's center (or, indeed, with any point in the wheel) (EN 68). It turns out that the lengths of the lines drawn by any point on the

[19] Descartes to Mersenne, 28 January 1641, AT III 293, translation in Ariew, R. "The Infinite in Spinoza's Philosophy" in *Spinoza: Issues and Directions*; The Proceedings of the Chicago Spinoza Conference [September 1986] Vol. 14, eds. Curley, E. and Moreau, P. F. (Leiden, New York: Brill's Studies in Intellectual History, 1990), pp. 16–31, on p. 17.

wheel's radius as it revolves are equal. Since even the line drawn by the center of the circle is equal to a line drawn on a radius length at infinity, the example illustrates the paradoxical result that the smallest number is equal to an infinite one (as they all yield lines of equal length).

Galileo then provides another arithmetical argument to make a similar point. This argument, which Leibniz cites in several places,[20] is referred to in the recent literature as "Galileo's paradox". It runs as follows: "all numbers, comprising of the squares and the non-squares, are greater than the squares alone" (EN 78; LLC 356). In other words, the series 1, 2, 3, 4, 5, 6, … (of squares and non-squares) has members that the series of squares alone 1, 4, 9, 16, … does not have (2, 3, 5, 6,). But, "there are as many square numbers as there are their own roots, since every square has its own root, and every root its own square…. But, if I were to ask how many roots there are, it cannot be denied that there are as many as all the numbers, because there is no number that is not a root of some square. That being so, it must be said that the square numbers are as many as all the numbers, because they are as many as their roots, and all numbers are roots" (EN 78; LLC 356). On the one hand, there appear to be more numbers than squares, but, on the other hand, there are as many numbers as squares. Thus, it turns out that the quantity of squares is both "less than" and "equal to" the quantity of all numbers – a paradox. Given this paradox, one might be inclined to infer that the relations of "greater than", "less than", and "equal to" do not apply in the context of infinity. This was Galileo's conclusion.

The radical conclusion Galileo draws from this argument is reiterated by Salviati: "from your ingenious argument we are led to conclude that the attributes 'larger,' 'smaller,' and 'equal' have no place either in comparing infinite quantities with each other or in comparing infinite with finite quantities" (EN 80). As Galileo writes, "I believe that these attributes of greatness, smallness, and equality do not befit infinities, about which it cannot be said that one is greater than, smaller than, or equal to one another" (EN 77–78; LLC 355).

Galileo concludes that insurmountable paradoxes arise when the notion of infinity is regarded as a quantity. The paradoxes he points to show that the most basic properties that must pertain to a quantity (such as "bigger than", "smaller than", or "equal to") do not hold in the case of "infinite quantity" (EN 80). As Knobloch has stressed, if this is the case, infinities should not be regarded as *quanta* at all.[21]

Leibniz's conclusion from his readings of Galileo's *Two New Sciences* in 1672–3,

[20] E.g. "Pacidius to Philalethes"; LLC 179.

[21] Knobloch, E., "Galileo and Leibniz: Different Approaches to Infinity," *Archive for the History of the Exact Sciences*, 54 (1999): 87-99.

however, was rather different. Instead of determining that the infinite does not belong in the realm of quantity, Leibniz comes to the conclusion that the notion of *an infinite number*, *seen as a whole*, is impossible. It is impossible precisely because such a notion violates the axioms of the realm of quantity, more specifically, the axiom stating that the whole is greater than its part.[22]

Leibniz argues that one cannot accept the result that the series of natural numbers is equal to the series of their squares; for, if this were permitted, the whole (the series of natural numbers) would not be greater than its parts (the series of squares). However, he finds "it difficult to agree" with Galileo's conclusion that the "appellations of greater, equal, and less have no place in the infinite" (A 6.3 551; LLC 179), "[f]or who would deny that number of square numbers are contained in the number of all numbers. But to be contained in something is certainly to be a part of it, and I believe it to be no less true in the infinite than in the finite that the part is less than the whole" (A 6.3 551; LLC 179).

Richard Arthur nicely presents the choices Leibniz sees as emerging from Galileo's paradox in the form of the following dilemma:

> "[Leibniz] identifies two candidates for rejection: (W) that in the infinite the whole is greater than the part, and (C) that an infinite collection (such as the set of all numbers) is a whole or unity . . . Leibniz upholds W, and this leads him to reject C. Cantor upholds C, and this leads him to reject W." (Arthur, 2001[23], pp. 103–104).

Leibniz concludes that the whole is greater than the parts even for the infinite, and therefore must deny that an infinite number is a whole. Thus, according to Leibniz, there cannot be a number of all numbers, or an infinite number. This implies that an infinite collection of elements cannot be regarded as a genuine unity.

As noted, the conclusion Galileo draws from his paradoxes is that, if the most basic relations of quantity do not hold in the realm of infinity, then infinity cannot be regarded as a quantity. This further implies that the finite and the infinite belong to different categories that cannot even be compared. Knobloch (1999) has put this point as follows. He argues that, according to Galileo, "[A]n 'infinite quantity' would . . . be a 'contradiction in terms', because an infinite would lack precisely those properties that characterize a quantity" (p. 94).

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[22] "[J]just as the proposition 'the whole is greater than the part' is the basis of arithmetic and geometry, i.e., of the sciences of quantity, similarly, the proposition 'nothing exists without reason' is the foundation of physics and morality, i.e., the sciences of quality, or, what is the same (for quality is nothing but the power of acting and being acted on) the sciences of action, including thought and action" (*Confessio* 35; A 6.3 118). See also Knobloch (1999), p. 94.

[23] Arthur, R. T. W., "Leibniz on Infinite Number, Infinite Wholes and the Whole World: A Reply to Gregory Brown", *Leibniz Review* 11 (2001):102-16.

Knobloch maintains that Leibniz's response to Galileo was to show, through his mathematical work on the calculus, that infinities can be handled in quantitative and precise terms. However, as I will argue below, while Knobloch's account is correct, it leaves out an important part of the story. Knobloch observes that Leibniz's calculus and his interpretation thereof show how the infinite can be dealt with mathematically, i.e., as a quantity. This is no doubt true. However, this observation holds only for *one* sense of infinity, which Leibniz (not surprisingly) reserves for quantities, numbers, and magnitudes (and for which he develops his syncategorematic approach). But there is another sense of infinity for which Leibniz actually accepts Galileo's position that the infinite cannot be regarded as a quantity. This is the notion of infinity that he would apply to being in general, and to God's being (and perfection) in particular. I will argue that this distinction, between a quantitative and a non-quantitative sense of infinity, is of great consequence for Leibniz's resolution of the paradoxes of infinity and for his wider metaphysics.

To a large extent, Leibniz's approach to infinity can be seen as a complex response to, and a sophisticated development of, Galileo's conclusions. In working his way through Galileo's *Dialogues*, Leibniz has "acquired" two challenges. (i) On the one hand, his recognition of Galileo's paradox motivates him to distinguish between a kind of infinity that he regards as non-quantitative and applicable to beings (and especially to the most perfect Being) and a kind of infinity that is quantitative and applicable to the mathematical domain. (ii) This immediately sets another challenge for Leibniz: to show how one can treat the notion of infinity within mathematics, that is, in a quantitative sense. Much of Leibniz' work on infinite series and the calculus during his Paris years can be seen as a response to this task. One result of his efforts is the syncategorematic interpretation of infinite terms (seen as fictions) that was presented in detail by Richard Arthur.[24]

At the same time, Leibniz's resolution of Galileo's paradox in terms of rejecting infinite number gives rise to another problem. The problem is how to account for the difference between the notion of an infinite number (which he regards as impossible) and the notion of an infinite being (the primary and most perfect being), which he regards not only as possible, but that it also implies a necessary being – one whose non-existence is impossible. How to account for the difference between these two notions is what I call "Leibniz's problem".

Leibniz's Problem: Infinite Being and Infinite Number

Leibniz's claim that "the number of all numbers is a contradiction" (e.g., A 6.3 463; DSR 7) appears in his Paris notes from 1675–76, a period in which he was developing his

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[24] See his introduction to LLC and Arthur R. T. W., "Leibniz's Syncategorematic Infinitesimals, Smooth Infinitesimal Analysis, and Second Order Differentials", *Archive for History of Exact Sciences*, 67 (April 2013): 553–593.

views about infinity in various domains. At the same time, Leibniz was also engaged, among many other projects, in distinguishing between possible and impossible notions. It is well known that Leibniz's view of possibility plays a central role in his metaphysics.[25] As early as his "Confession of a philosopher" (1672–73), Leibniz defines a possibility such that *x* is possible if it has a notion whose internal constituents are consistent. In this context, Leibniz is using the notions of the "number of all numbers" (*numerum omnium numerorum*) and the "greatest or maximal number" (*numerus maximus*) as an illustration of an impossible notion, that is, a notion whose internal constituents imply a contradiction. In the same texts, Leibniz also uses the notion of the number of all numbers in contrast to a notion whose possibility he is keen to prove – that of "the greatest or the most perfect being" (A 6.3 572; DSR 91).

Comparing the notion of the greatest being with the notion of the greatest number gives rise to a severe problem. Leibniz states this problem in a letter to Oldenburg from December 1675:

> "Whatever the conclusions which the Scholastics . . . and others derived from the concept of that being whose essence is to exist, they remain weak as long as it is not established whether such being is possible, provided it can be thought. To assert such a thing is easy; to understand it is not so easy. Assuming that such a being is possible or that there is some idea corresponding to these words, it certainly follows that such a being exists. But we believe that we are thinking of many things (though confusedly) which nevertheless imply a contradiction; for example, the number of all numbers. We ought strongly to suspect the concepts of infinity, of maximum and minimum, of the most perfect, and of allness (*omnia*) itself. Nor ought we believe in such concepts until they have been tested by that criterion which must, I believe, be credited to me, and which renders truth stable, visible and irresistible." (GM I 83-84; L 257)

The worry raised by Leibniz here is made even clearer in a letter to Countess Elizabeth, written three years later (1678). There, Leibniz considers several examples of impossible notions (such as those of the squared circle and of the greatest speed) and writes,

> "we think about this greatest speed, something that has no idea since it is impossible. Similarly, the greatest circle of all is an impossible thing, and the number of all possible units is no less so; we have a demonstration of this. And nevertheless, we think about all this. That is why there are surely grounds for wondering whether we should be careful about the idea of the greatest of all beings, and whether it might not contain a contradiction." (A 2.1 433–38; AG 238)

Leibniz's reasoning here is very clear. Since we entertain thoughts about things such as the greatest speed and the greatest number, which upon analysis prove to be contradictory, we ought to examine whether the idea of the greatest of all beings might not turn out to be

[25] For more details regarding Leibniz's view of possibility, see reference to Author.

contradictory as well. In fact, we often use concatenations of words that do not correspond to any idea and which might well turn out to be contradictory.

Leibniz's problem is therefore to show that, while the greatest number is contradictory and thus impossible, the greatest or most perfect being, i.e., God, is not.

The way in which Leibniz compares and contrasts the notions of infinite number and that of the infinite (or most perfect) being has drawn surprisingly little attention from scholars. Even more importantly, the significance of this comparison for Leibniz's views about infinity has gone almost unnoticed. This problem, however, is central for understanding Leibniz's complex approach to infinity and the context in which it develops. Formulating this dilemma in detail will set the stage for exploring Leibniz's solution to this problem through his encounter with Spinoza.

Leibniz's Encounter with Spinoza

Leibniz and Spinoza

In his years in Paris, and particularly in 1675 and 1676, Leibniz shows immense curiosity and interest in Spinoza's philosophy, which he often discusses with his friend Tschirnhaus. His attempts to obtain the secretly guided manuscript of *Ethics* via Tschirnhaus fails. But he does get to read and annotate Spinoza's letter to L. Meyer in which Spinoza lays out his views on infinity (Ep. 12). Leibniz read and annotated this letter in April 1676.[26] In 1676, Leibniz reluctantly traveled from Paris back to Hanover. He made a point to travel via the Hague in order to meet Spinoza. The two philosophers met. But this was their only meeting. Their philosophical systems, however, have many more meeting points. In the Hague, Leibniz showed Spinoza his modified version of Anselm's (and Descartes') proof for the existence of God. According to this version of the argument, the notion of the *Ens Perfectissimum* entails existence; for it includes all perfections, and existence is considered a perfection. Leibniz found this reasoning unsatisfactory for the following reason: one needs to show not only that the conclusion follows from the premises, but also that the definition of the *Ens Perfectissimum* as the subject of all perfections is consistent – a point that was taken for granted by all previous upholders of the argument. In other words, Leibniz argues that, in order to prove that a most perfect being exists, one has to show first that this notion

[26] See Lærke , M. *Leibniz lecteur de Spinoza. La genése d'une opposition complexe* (Paris: Honoré Champion, 2008), pp. 423–24. This letter is not the only source Leibniz obtains regarding Spinoza's views at the time. He receives quite accurate information on the *Ethics* from Tschirnhaus, with whom he discusses Spinoza's metaphysics as well as questions of mathematics (see for example Leibniz's letter of May 1678, GM IV 451–63; L 294–99).

is consistent, that is, that it is possible (A 6.3 572; DSR 91; A 6.3 583; DSR 105-07).[27]

The issue Spinoza and Leibniz discussed in their meeting in The Hague is highly indicative of some of the remarkable affinities, as well as some of the deep rifts, between their views regarding the nature of the infinite, and especially the relation between the notions of infinite being and infinite number. While Leibniz's approach implies that the existence of an infinite and most perfect being follows from its essence, Spinoza holds that, since being finite involves some negation, infinity expresses (or, one might say, is) the very absolute affirmation of existence.[28]

Another striking difference in their metaphysical systems is that, for Spinoza, there is only a single substance, whereas Leibniz speaks of infinitely many beings. As it is usually put, Spinoza is a substance monist and Leibniz a substance pluralist. Indeed, this distinction captures a major difference between Spinoza's and Leibniz's metaphysical systems. Yet a close analysis reveals some fundamental agreements regarding the claim that any substance is, by definition, both infinite and unique. In light of this similarity, I will suggest that, when Leibniz and Spinoza say that the divine substance is infinite, it is to be understood primarily in a non-quantitative sense.[29]

Spinoza's Letter and Leibniz's Response

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[27] Cf. a note from 1676, in which Leibniz writes, "[i]n the chapter of St. Thomas' *Summa Contra Gentiles* which is entitled 'Whether the existence of God is known *per se*,' there is a reference to an elegant argument which some use to prove the existence of God. The argument is: God is that than which nothing greater can be thought. But that than which nothing greater can be thought cannot not exist. For then some other thing, which cannot not exist, would be greater than it. Therefore God cannot not exist. This argument comes to the same as one which has often been advanced by others: namely, that a most perfect being exists. St. Thomas offers a refutation of this argument, but I think that it is not to be refuted, but that it needs supplementations. For it assumes that a being which cannot not exist, and also a greatest or most perfect being, is possible" ("On Truths, the Mind, God, and the Universe" 15 April, 1676, A 6.3 510–511; DSR 63).

[28] "Since to be finite is some negation and to be infinite is an absolute affirmation of the existence of some nature, it therefore follows from proposition 7 that any substance must necessarily exist" (EIP8S). Spinoza argues further that one can adequately consider the uniqueness of being (i.e., of substance) with respect to its existence alone and not its essence. As Spinoza writes to Jarig Jelles, "in an *Appendix to the Principles of Descartes, Geometrically Demonstrated* I established that God can be called one [*unum*] or unique [*uniqum*] only in a very inappropriate sense, I respond that a thing cannot be called one and unique with respect to essence but only with respect to existence. We conceive of things as existing in a certain number of exemplars only if they are brought under a common genus" (Ep. 50 to Jarig Jelles, Gebhardt IV 239). I take this to imply that, according to Spinoza, one cannot *conceive* of the unique and infinite being in abstraction from its existence, as a pure essence. In addition, one may talk about the unique existing thing in a numerical sense only in an inappropriate sense. For the category of number can only apply to things that can be "brought under a common genus", which obviously does not hold of a unique being.

[29] The thesis of the non-quantitative sense of infinity, as presented below, also provides a partial explanation for why Leibniz was attracted to Spinoza's philosophy during his years in Paris (especially in 1675–76) and, at the same time, why he ultimately moved away from it while reading and commenting on Spinoza's *Ethics* in 1678. For details of the complex way in which Leibniz read Spinoza, see Lærke (2008).

Like Leibniz, Spinoza too has to account for the difference between the infinity of number and the infinity of God. Spinoza explicitly defines God as "a being absolutely infinite, that is, a substance consisting of an infinity of attributes" (EID6).[30] In Ep. 12, Spinoza takes the following approach to this problem: he distinguishes between different kinds of infinity and, in particular, between a kind of infinity that applies to (indivisible) substance and a kind of infinity that applies to divisible quantities. This approach emerges explicitly when Spinoza discusses the nature of the infinite and how to dissolve the traditional paradoxes surrounding it.

Spinoza's argumentation in this letter is of considerable complexity. However, one point is rather clear: Spinoza holds that the notion of infinity that may apply to the substance is non-quantitative. Since Spinoza identifies God with the unique substance, God's infinity is not comparable to that of numbers. The reason is that any reference to numbers presupposes a limitation and hence would imply that it is finite. According to Spinoza, the tendency to describe a substance with numerical infinity is entirely misguided and generates contradictions. The way out from the contradictions affecting the infinite is to avoid the common confusion between the *quantitative sense* of infinity that can adequately be ascribed to number and divisible quantities, and the sense of infinity that can adequately be ascribed to a unique and indivisible substance.

It is worth noting that Leibniz's annotations on Spinoza's letter begins by stating that Spinoza "demonstrates that every substance is infinite, indivisible, and unique" (A 6.3 275; LLC 101). Leibniz then copies (almost to the letter) Spinoza's definitions of substance (EID3) and of God (EID6). This is certainly indicative of the interest Leibniz takes in reading this letter. Particularly indicative here is Leibniz's addition to Spinoza's definition of God.

> "*He defines God as follows:* that which is an *absolutely* infinite being, i.e. a substance consisting of infinite attributes, each of which expresses an infinite and eternal essence and is thus immense [*immensum*]." (LLC 103)[31]

The clause "*adeoque immensum est*" is not part of Spinoza's definition but is added by Leibniz. This is telling. In his annotations to this letter (A 6.3 282; L 24; LLC 115), Leibniz states the following: "I have always distinguished the *Immensum* from the *Interminato*, i.e., that

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[30] The translation is significant here. It can also be translated "consisting of infinite attributes". This is important in identifying the kind of infinity that is at work here. In addition, this plays into the debate regarding the number of attributes that God is said to have. Those who translate it as "an infinity" tend to hold that there is a numeric infinity of attributes, while "infinite attributes" is related to the infinite nature of the attributes. I prefer the former but have opted to using Curely's translation throughout.

[31] « *Deum sic definit.* Quod sit Ens *absolute* infinitum, hoc est substantia constans infinitis attributis, quorum unumquodque infinitam et aeternam essentiam exprimit adeoque immensum est » (A 6.3 276).

which has no bound [*seu terminum non habente*]." In notes from this period, Leibniz is using *Immensum* as a noun – the *Immensum* – designating God as infinite but beyond measure. He also uses *Immensum* as "that which persists during continuous change in space . . . and is one and indivisible" (A 6.3 519, see LLC 450). Evidently, Leibniz is using the notion of *Immensum* in more than one sense. Likewise, divine immensity is taken as the "basis of space" (ibid). However, it seems clear that, unlike the current English connotations of the word "immense", Leibniz does not use *immensum* here to indicate large or immense *magnitude;* rather, he uses it in a way much closer to its literal meaning in Latin, that is, to indicate something *beyond any measure*, or more precisely, something that has no measure (and is therefore impossible to measure) – something that *cannot* be measured because it does not belong to the category of quantity.

Both recent English translators, Parkinson and Arthur, have emphasized this point (see DSR 122, n. 92 and LLC 450). To avoid the current English connotations of "immense", Parkinson renders *immensum* as "immeasurable", so that the Latin negation of measure (*mensura*) is conspicuous in the translation. As Arthur notes in the Glossary to his edition, "*Immensum* can be synonymous with 'infinite' or 'beyond measure', as Leibniz employs it in Aiii4: 95; and at Aiii60: 475, where Leibniz distinguishes this species of the infinite from the unbounded." (LLC 450, Latin–English Glossary)

Thus, when Leibniz adds his gloss to Spinoza's definition of God, i.e., that the absolutely infinite being is also *immensum*, he refers to one of his own notions of infinity, viz., that which is beyond measure. It is in this sense that *immensum* is distinguished from the unbounded. The unbounded infinite designates a *measurable* quantity, whereas *immensum* designates something that *cannot be measured.* Thus, Leibniz wishes to emphasize that God is something beyond any measure – something that cannot to be described in quantitative or measurable terms and probably inadequately described in quantitative terms. To recap: the main point here is that Leibniz's addition indicates that, in his eyes, the infinity of the divine substance cannot be quantified or measured but rather belongs to an altogether different category.[32]

Leibniz then adds a very interesting note on a being conceived through itself (*per se concipi*):

> "we understand through itself only that which is its own cause, i.e., that which is necessary, i.e., is a being in itself. And so it can be concluded from this that if we understood a necessary being, we would understand it through itself. But it can be doubted whether we do understand a necessary being, or, indeed, whether it could be understood [*intelligatur*] even if it were known or recognized [*cognosci*]." (A 6.3 275; LLC 101)

[32] For a slightly different emphasis on Leibniz's addition of the word *immensum*, see Lærke (2008), pp. 469–477 and pp. 424–25.

In reading Spinoza's letter, Leibniz seems to recall the difficulty of showing that the notion of a necessary being can be understood or, in other words, that it is intelligible. According to Leibniz, in order to show that something is intelligible, one has to show how it is produced i.e., by showing that its concept is consistent. Thus, it seems that, in reading Spinoza's letter, Leibniz is still occupied with his own problem as well.

Given Leibniz's preoccupation with the tension between the possibility of an infinite being and the impossibility of an infinite number, it is not surprising that he is interested in the way that Spinoza connects the definitions of substance, God, and infinity. He agrees with Spinoza that any substance "is infinite, indivisible, and unique." Yet, according to him, the possibility of such a being needs to be demonstrated. While he maintains his demand to prove the *possibility* of the most perfect being, Leibniz does not restate (but only mentions) his proof after 1676.[33] As we saw in the previous section, Leibniz demonstrates that an infinite collection of discrete units is impossible and cannot be regarded as a whole. At the same time, he clearly regards God as an infinite unity. In fact, he calls God the one-all (*unus omnia,* A 6.3 385), and maintains that such a being is possible.

In light of this, one can reasonably suppose that Leibniz would seek support for his line of reasoning regarding the possibility of an infinite being and the impossibility of an infinite number. Such support may indeed be found in Spinoza's letter. Towards the beginning, Spinoza notes that,

> "everyone has always found the problem of the Infinite very difficult. Indeed insoluble.[34] This is because they have not distinguished between what is infinite as a consequence of its own nature, or by the force of its definition, and what has no bounds, not indeed by the force of its essence, but by the force of its cause. And also because they have not distinguished between what is called infinite because it has no limits and that whose parts we cannot explain or equate with any number, though we know its maximum and minimum. Finally, they have not distinguished between what we can only understand, but not imagine, and what we can also imagine." (LLC 103; Curley 201)

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[33] In his *Leibniz: Determinist, Theist, Idealist* (1994), Adams has argued convincingly that the a priori proof for the possibility of the notion of the *Ens Perfectissimum* gives way to a presumption of its possibility. See Chapter 8, and A 2.1 436 for an explicit text endorsing the presumption of possibility. Lærke (2008) notes that, after 1677, Leibniz only mentions his *a priori* proof but it never appears in his later writings. I am not sure what conclusion should be drawn from this. It is obvious that Leibniz maintains that the *Ens Perfectissimum* is possible. However, it is not obvious on what grounds. From the fact that he does not repeat the argument, it cannot be concluded that he abandoned it, as Lærke seems to hold. The presumption of possibility might be an addition to, rather than a replacement for, the *a priori* argument. As far as I can see, we simply cannot tell.

[34] In his exposition of Descartes' *Principles of Philosophy*, Spinoza mentions some of the traditional difficulties associated with the infinity: "if an infinite is not greater than another, quantity A will be equal to its double, which is absurd"; "whether half an infinite number is also infinite, whether it is even or odd, and the like" (in Gebhardt I 190. also see 192–196); Also see Ariew, 1990, pp. 16-31.

He adds:

> "If they have attended to these distinctions, I maintain that they would never have been overwhelmed by such a great crowd of difficulties. For then they would have understood clearly what kind of Infinite cannot be divided into any parts, or cannot have any parts, and what kind of Infinite can, on the other hand, be divided into parts without contradiction. They would also have understood what kind of Infinite can be conceived to be greater than another Infinite, without any contradiction, and what kind cannot be so conceived." (LLC 103–105; Curley 202)

According to Spinoza, different kinds of infinity that correspond to different kinds of things (viz., Substance, attributes, and modes).[35] By means of these distinctions, Spinoza qualifies and restricts the way in which infinity can be ascribed to substance. Spinoza's distinction suggests an attractive approach to Leibniz's problem. According to Spinoza, one kind of infinity (the one pertaining to infinite being) "cannot be divided into any parts, or cannot have any parts", and the other kind of infinity (the one pertaining to modes) "can... be so divided into parts without contradiction." This is of course related to Spinoza's view that, strictly speaking, a substance is infinite and indivisible (EIP15S). For this reason, a substance is not divided into parts; rather, its attributes have various modes.

According to Spinoza, the kind of infinity that we can ascribe to substance is such that "we cannot explain or equate with any number". An infinite substance, on this view, is non-divisible and cannot be understood in numerical terms. For this reason, the use of this kind of infinity would not involve the contradictions that affect things whose enumeration requires comparison and abstraction by the imagination. In fact, Spinoza maintains that enumeration involves abstraction and comparison of things under a common genus by means of the imagination.[36] However, the kind of infinity that pertains to a substance cannot even be adequately conceived by the imagination but only by the intellect. This can clearly be seen in the following passage from Ep. 12 (which summarizes EIP15S):

> "we conceive quantity in two ways: either abstractly, or superficially, as we have it in the imagination with the aid of the senses; or as substance, which is done by the intellect alone. So if we attend to quantity as it is in the imagination, which is what we do most often and most easily, we find it to be divisible, finite, composed of parts, and one of many. But if we attend to it as it is in the intellect, and perceive the thing as it is in itself, which is very difficult, then we find it to be infinite,

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[35] Duration, number, and motion are seen as mere auxiliaries of the imagination, which serve as measures of divisible magnitudes. Cf. Gueroult M., *Spinoza I: Dieu* (Paris: Aubier-Montaigne, 1968), Ariew 1990, as well as Lærke 2008.

[36] See EIP15, Ep. 34, Ep. 50, and the next section for more details.

> indivisible and unique, as I have already demonstrated sufficiently to you before now." (A 6.3 278; LLC 107; Curley 202–203)[37]

This last point – which Leibniz mentions in the first line of his annotations to Spinoza's letter – suggests a way out of the inconsistency he identifies in infinite quantity. In line with Spinoza's reasoning, Leibniz can distinguish between "beings" and "non-beings" by observing that each requires a different kind of infinity. And this would account for his regarding an infinite being as possible and an infinite number as impossible. At the same time, the concept of an "infinite being" is to be taken in a syncategorematic sense (or more precisely, the term 'infinite' in the *concept* of an infinite being).

Thus, what's most pertinent for Leibniz's purposes in Spinoza's letter can be paraphrased as follows: any number is by definition limited. For this reason, it is also measurable. By contrast, God's infinity cannot be quantified, measured, or numbered, precisely because this would imply limiting it (or seeing it as limited), as well as viewing God as a divisible and discrete entity, which Spinoza clearly regards as absurd. This suggests that, for Spinoza, "infinity" is used differently when ascribed to numbers (or more generally to divisible and discrete quantities, or to a feature of modes and abstractions) and when it applies to the all-inclusive substance or God. A substance is said to be infinite on account of its completeness and absolute perfection. Therefore, for Spinoza, it must be *in*divisible and admit of no parts. In this sense, a substance is said to be infinite in a non-quantitative sense.

Given the context presented in the first section, it should now become clear why Leibniz would be receptive to such a view. Indeed, he seems to agree with Spinoza's analysis. Yet, as is typical of him, Leibniz does not simply accepts Spinoza's analysis; rather, he recasts Spinoza's distinction in his own terms and appropriates it for his own purposes.[38] In his annotations, he writes,

> "I set in order of degree: *Omnia*; *Maximum*; *Infinitum*. Whatever contains *everything* is maximum in entity; just as a space unbounded in every direction is maximum in extension. Likewise, that which contains everything is most infinite [*infinitissimum*], as I am accustomed to call it, or the absolutely infinite. The *Maximum* is everything of its kind, i.e., that to which nothing can be added, for instance, a line unbounded on both sides, which is obviously also infinite; for it contains every length. Finally those things are *infinite in the lowest degree* whose magnitude is greater than we can

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[37] Cf.: "If therefore we consider quantity as it is in the imagination, that which is the most ordinary, we find that it is finite, divisible and composed of parts; if, on the contrary, we consider it as it is in the understanding and we conceive it insofar as it is substance, then, as we sufficiently demonstrated, we will find it to be infinite, unique and indivisible" (EIP15S).

[38] Obviously, I do not argue here for a direct influence in the sense that Spinoza's letter is the exclusive or even the main source for Leibniz's views on infinity. Rather, I claim that Leibniz's attraction to Spinoza's view is evident in his annotations and that his response to Spinoza's views is revealing of and serves him to articulate his own views.

> expound by an assignable ratio to sensible things, even though there exists something greater than these things. . . . For a maximum does not apply in the case of numbers." (A 6.3 282; LLC 114–15)[39]

As Lærke notes,

> "If one compares this classification with the one proposed in Spinoza's letter 12, one is struck by their similarity. First, the distinction between *maximum* and *omnia* evokes the distinction between the attributes, which are infinite "in their kind" in EIdef.4 and the 'absolutely infinite' substance in EIdef.6 – which is exactly the definition reproduced at the beginning of the *Communicata ex litters domini Schulleri*. [Likewise] there is a strong resemblance between that which Leibniz calls '*immensum*' and that which Spinoza calls 'infinite by nature'." (Lærke 2008, 433, my translation)

The similarity between *immensum* and "infinite by nature" is particularly remarkable. As Lærke also notes, "that which is infinite by nature or by virtue of its definition, is the substance" (ibid, 430). We have already seen that Leibniz amends Spinoza's definition of God with the clause "that which is *immensum*".

There are some significant differences between Leibniz and Spinoza here. The most conspicuous difference is that Leibniz reformulates Spinoza's distinction in terms of degrees. *Omnia*, he says, "is the highest degree, [it] is everything, and this kind of infinite is God, since he is all one; for in him are contained the requisites for existing of all the others" (A 6.3 385; LLC, 43). Elsewhere, and later in his career, Leibniz is also very clear that the highest degree, the "absolutely infinite", applies to God alone. For example, in a letter to Des Bosses, from 11 March 1706, he notes that "only indivisible and absolute infinite has true unity: it is God" (GP II 305). In the *New Essays* (2.17.1), he writes that "rigorously speaking, the true infinite is only in the absolute, which is anterior to any composition and is not formed by the addition of parts" (GP V 144).[40] This notion of absolute infinity is non-quantitative in the sense that God or the most perfect being has a non-divisible unity, which admits of no parts; also, it cannot be compared to or measured by any quantity. In this sense, absolute infinity indicated allness and perfection, which cannot be measured. Hence, this notion of infinity is aptly called the Immeasurable or *Immensum*. It involves absolute perfection, completeness and, most important to my concerns here, inherent unity and indivisibility.

As noted, this conception of the infinite would support Leibniz in avoiding the difficulty facing the notions of infinite number, line, speed, shape, or any other magnitude.

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[39] Compare with A 6.3 385; LLC 43, where Leibniz articulates the same threefold distinction in slightly different words.

[40] « *Le vrai infini à la rigueur n'est que dans l'absolu, qui est antérieur à toute composition, et n'est point formé par l'addition des parties* » (GP V 144; NE 2.17.1)

Simply stated, on such a conception of infinity, quantitative categories are inapplicable to true beings.[41] And likewise, maximal quantities cannot be regarded as perfections (see the "Discourse on Metaphysics", paragraph 1). Therefore, if infinity is ascribed to a substance not in a quantitative sense but only in the sense of absolute infinity*,* the notion of infinite substance or being, qualified in this way, would avoid the contradiction of infinite number and other infinite magnitudes.

While the numerical infinite does not constitute a complete being, infinity in the absolute sense of *Omnia,* does. Leibniz thus reserves the notion of the absolutely infinite for God or the most perfect being. Early in 1672 Leibniz observed that, "[t]here is no maximum in things, or what is the same, the infinite number of all unities is not one whole, but is comparable to nothing" (A 6.3 98; LLC 13). Even though their views about the divine substance are quite different, the connection between infinity and unity is crucial for both Spinoza and Leibniz. In this regard, Leibniz and Spinoza share the following view: substance is the only thing of which one can say that it is an infinite and unique and indivisible. As we have seen, however, this conception involves a non-quantitative understanding of the infinity of substance.

Conclusion

Neither Descartes nor Leibniz is shy about defining God as having infinitely many attributes. The same is true of Spinoza, who encounters a similar problem to Leibniz's. However, Spinoza's refusal to ascribe infinite number to God seems less problematic because, for him, questions of possibility are merely epistemic and merely betray a deficiency of knowledge. For Leibniz, however, consistent concepts indicate pure possibilities. Leibniz's possibility proof of 'a most perfect being', of which he takes pride throughout the rest of his life, turns on his observation that a subject that includes all positive perfections does form a consistent concept. But it is exactly such a concept that seems to suggest an infinite number of perfections. Leibniz proves that a being whose notion consists of infinitely many attributes is consistent, but a number whose notion consists of infinitely many units is not. Just as the notion of infinite number implies infinitely many units, the notion of God seems to imply infinitely many perfections or attributes.

If this were the case, however, Leibniz would have to consider both notions to be equally problematic. Yet, he clearly does not believe this to be the case. Rather, he considers the notion of an infinite being to be possible, while he considers the notion of an infinite number to be impossible. If the notions of infinite number, most rapid motion, and the greatest shape are all contradictory, as Leibniz holds, he has to show that the notion of the infinite being is not.

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[41] This is at least true in the case of God. The case of created substances is much more delicate.

Roughly stated, Leibniz's approach to the problem turns on the observation that an infinite being is infinite in a non-quantitative sense. Such a being is not a whole composed of discrete parts, nor, strictly speaking, does it admit of parts at all; rather, it is said to be indivisible and immeasurable in the sense that its perfection and being is not something that can be divides or quantified.

As we have seen, in the texts from 1675-6, Leibniz's view concerning the infinity of substance is very close to that of Spinoza. Moreover, Spinoza's view on infinity offers an attractive way to approach Leibniz's problem and clarify his approach regarding the way in which a substance may be said to be infinite. This approach turns on the distinction between two senses of infinity: one that can be quantified, and one that cannot.

With respect to the consistency problem, Leibniz's solution turns on stressing that both infinite number and the notion of an infinite being are *concepts*. In other words, concepts are not entities; certainly, Leibniz does not regard them as entities or true beings. If his syncategorematic solution works for the notion of an infinite number, it should work for the *concept* of the most perfect being as well. While a concept should not be seen as a unity, the entity itself must be seen as a unity. The concept indicates a possibility – and thus can be regarded in a quantitative sense; an entity is seen as a real, actual being and hence has to be regarded as infinite in a qualitative sense. This is the gist of the resolution I proposed above. It turns on distinguishing between different contexts of using infinity.

In fact, Leibniz's reading of Spinoza reveals more than this. As we have seen, he recasts Spinoza's distinction between kinds of infinity, each with a different domain of application, in terms of degrees. As he writes: "I set in order of degree: *Omnia*; *Maximum*; *Infinitum*." Roughly speaking, between the highest degree of infinity, which Leibniz clearly ascribes to the absolute and necessary Being, and the lowest degree of infinity, which he ascribes to *entia rationis* such as numbers and relations, Leibniz invokes a third, intermediate degree of infinity – a maximum of its kind. But Leibniz's use of this intermediate degree of infinity must be developed elsewhere.[42]

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[42] I expand on this in my book manuscript [reference to author]